FACULTY OF ENGINEERING AND ARCHITECTURE

Quantifying and minimizing risk of conflict in social networks

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Research Interest

- We are interested in ***opinions*** and ***conflict*** in ***social networks***.

How do you form your opinions?

- Friedkin and Johnsen's opinion formation model.
- People have two different kinds of opinions:
  - internal opinions **s**
  - expressed opinions **z**

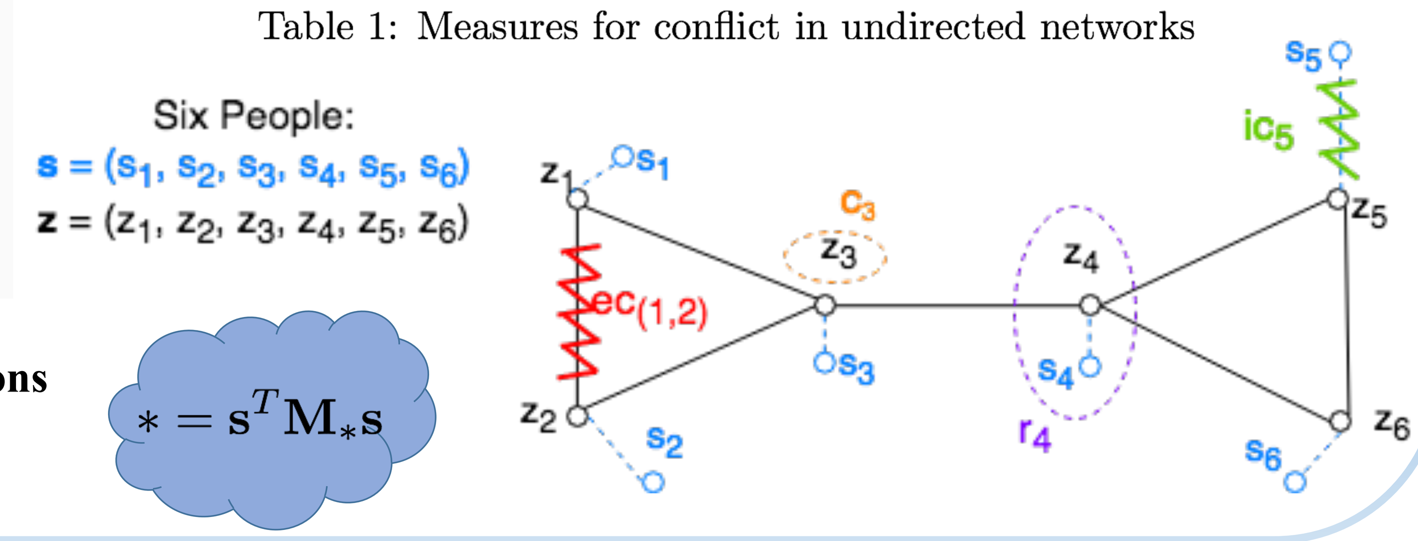
$$z_i = \frac{s_i + \sum_{j \in N(i)} w_{ij} z_j}{1 + \sum_{j \in N(i)} w_{ij}} \quad \Longrightarrow \quad \mathbf{z} = (\mathbf{L}+\mathbf{I})^{-1}\mathbf{s}$$

Figure 1: Form opinions through social interactions with others with potentially differing opinions.

How to measure the conflict?

Name	Definition	s
Internal Conflict: ic	$\sum_i (z_i - s_i)^2$	$\mathbf{s}^T (\mathbf{L}+\mathbf{I})^{-1} \mathbf{L}^2 (\mathbf{L}+\mathbf{I})^{-1} \mathbf{s}$
External Conflict: ec	$\sum_{(i,j)\in E} w_{ij} (z_i - z_j)^2$	$\mathbf{s}^T (\mathbf{L}+\mathbf{I})^{-1} \mathbf{L} (\mathbf{L}+\mathbf{I})^{-1} \mathbf{s}$
Controversy: c	$\sum_i z_i^2$	$\mathbf{s}^T (\mathbf{L}+\mathbf{I})^{-2} \mathbf{s}$
Resistance: r	$\sum_i s_i z_i$	$\mathbf{s}^T (\mathbf{L}+\mathbf{I})^{-1} \mathbf{s}$

Table 1: Measures for conflict in undirected networks



$$* = \mathbf{s}^T \mathbf{M}_* \mathbf{s}$$

Conflict does not go away…

$$ic + 2ec + c = \mathbf{s}^T \mathbf{s}$$

(A conservation law of conflict)

- **Challenges in existing work**
  - Getting **s** is beyond reach in practice.
  - Minimize the conflict on one issue may increase conflict on another.
- **Problem Definition**

  *Given a social network G = (V, E), how to minimize the conflict without knowing people's opinions? (i.e., without **s**)*

Our approach:

Focus on **risk of conflict**, which is independent of any particular set of opinions, depending purely on the topology of the network.

- **Conflict Risk of a Network**
  - **Average-case Conflict Risk (ACR)** $\quad (E[\mathbf{s}\mathbf{s}^T] = \mathbf{I})$

$$\mathrm{ACR}_* = E[\mathbf{s}^T \mathbf{M}_* \mathbf{s}] = E[\mathrm{trace}(\mathbf{s}\mathbf{s}^T \mathbf{M}_*)] = \mathrm{trace}(E[\mathbf{s}\mathbf{s}^T]\mathbf{M}_*) = \mathrm{trace}(\mathbf{M}_*).$$

  - **Worst-case Conflict Risk (WCR)**

$$\mathrm{WCR}_* = \max_{\mathbf{s}\in\{-1,1\}^n} \mathbf{s}^T \mathbf{M}_* \mathbf{s}$$

How to minimize the risk of conflict?

$\mathbf{s} \to \mathbf{S} \in \{-1,1\}^{n\times \ell}$

Consider a set of worst-case **s** instead of just a single one. **More Robust.**

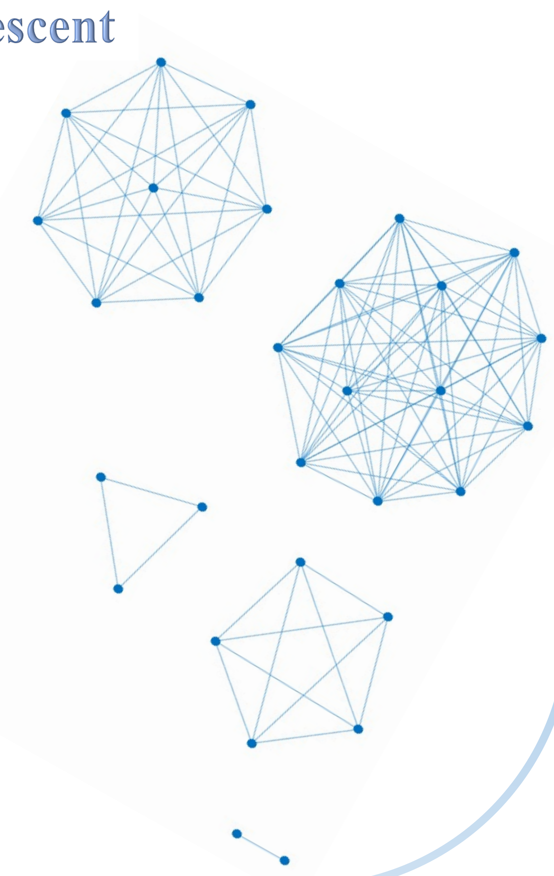
- **Optimization Problems**
  - **Optimize ACR**

$$\min_{\mathbf{A}} \mathrm{trace}(\mathbf{M}_*), \quad \text{s.t. } 0 \le \mathbf{A} \le 1, \quad \mathrm{sum}(\mathrm{sum}(\mathrm{abs}(\mathbf{A}-\mathbf{A}_0))) \le 2k.$$

  - **Optimize WCR**

$$\min_{\mathbf{A}} \max_{\mathbf{s}\in\{-1,1\}^n} \mathbf{s}^T \mathbf{M}_* \mathbf{s}, \quad \text{s.t. } 0 \le \mathbf{A} \le 1, \quad \mathrm{sum}(\mathrm{sum}(\mathrm{abs}(\mathbf{A}-\mathbf{A}_0))) \le 2k,$$

- **Conditional gradient descent versus Coordinate descent**
  - Conditional gradient descent: It seeks a step most aligned with the gradient, while respecting the constraints after taking a finite step along the direction.
  - Coordinate descent: It takes the step on only one edge that leads to the largest decrease of the objective value.



Theoretical finding:

- **Local optima for the ACR: complete (sub)graphs**

ec $\quad$ *c* $\quad$ ***r* – global optima**

Experimental findings:

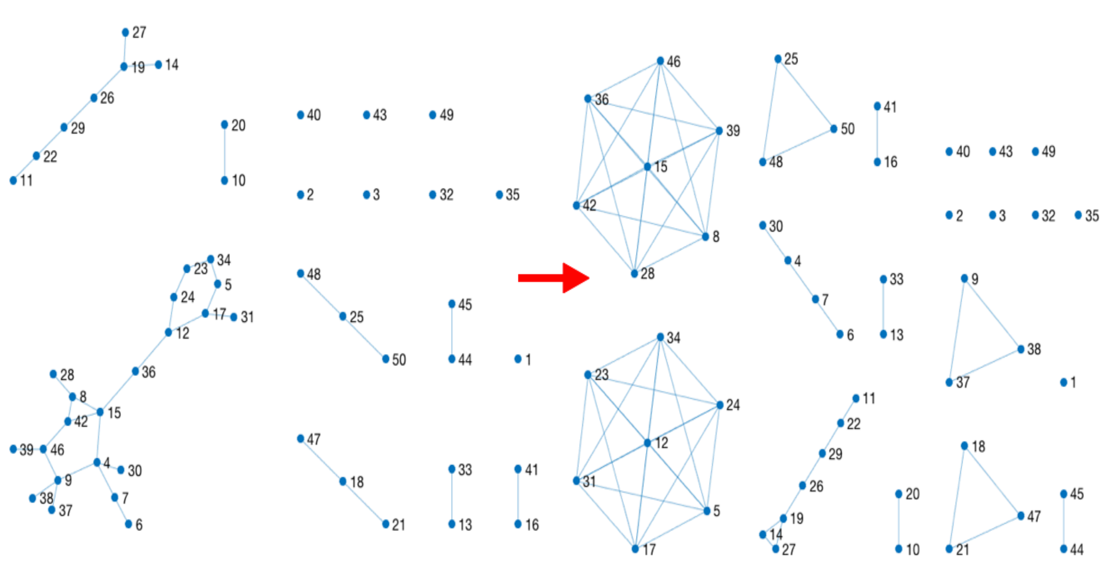


Figure 2: Optimization for the ACR of external conflict *ec* on an Erdős-Rényi network (*n*=50, *p*=0.03) using conditional gradient descent with *k*=2.

External Conflict (*ec*), is arguably the most relevant among the conflict measures in practice, also exhibits the most complex behavior.

*Interestingly, the structures at the local optima of the ACR for ec seem to correspond with **common management structures in companies**: a flat organization corresponds to a clique, while a hierarchical organization corresponds to a tree. Management practice may well have evolved this way in part because it minimizes conflict.*

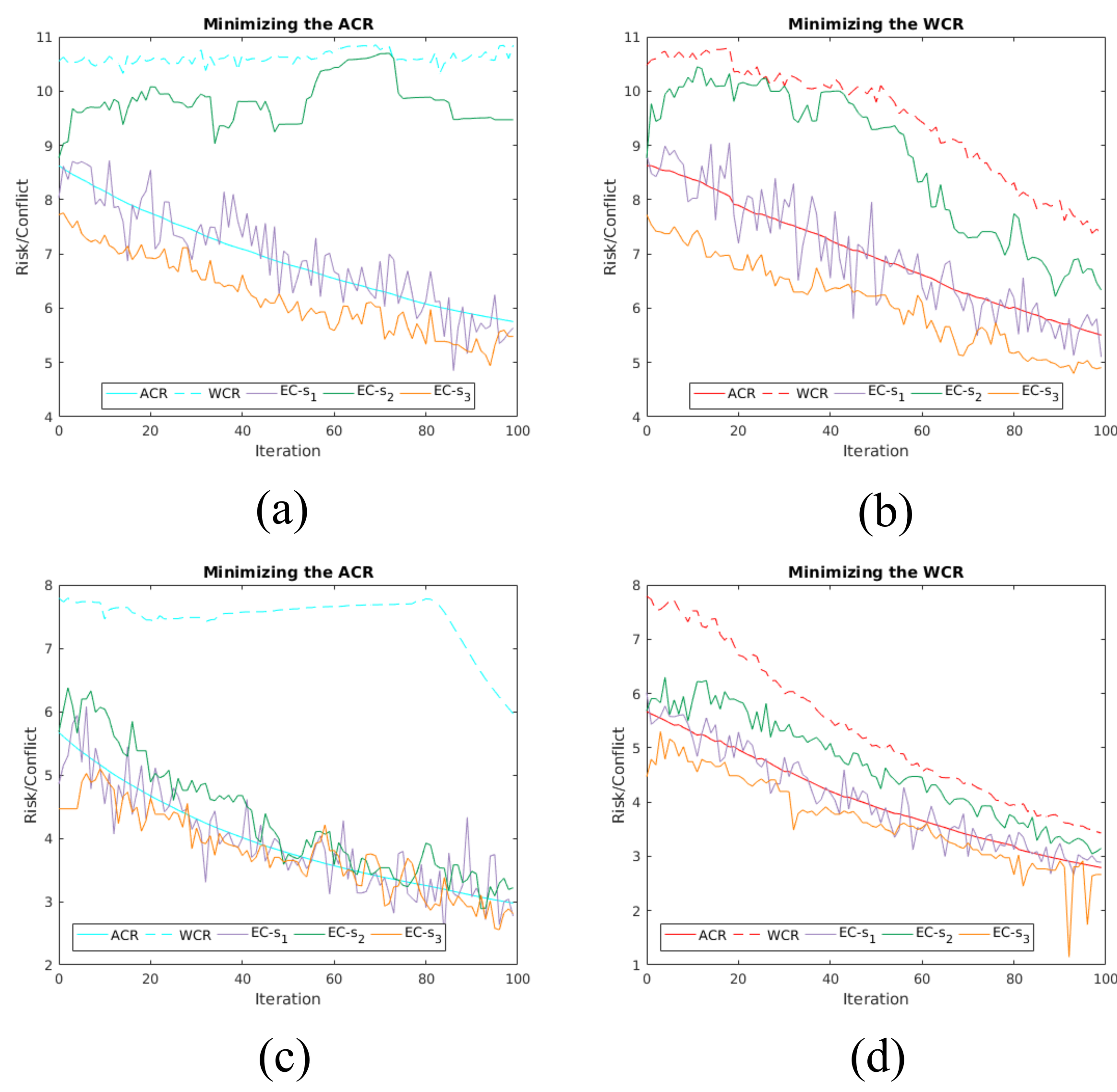


Figure 3: The ACR, WCR, and actual conflict for three internal opinion vectors over consecutive iterations with respect to *ec*. (a), (b) are based on an ER model (*n* = 50, *m* = 60) with conditional gradient descent $k = 1$; (c), (d) on Karate with coordinate descent.

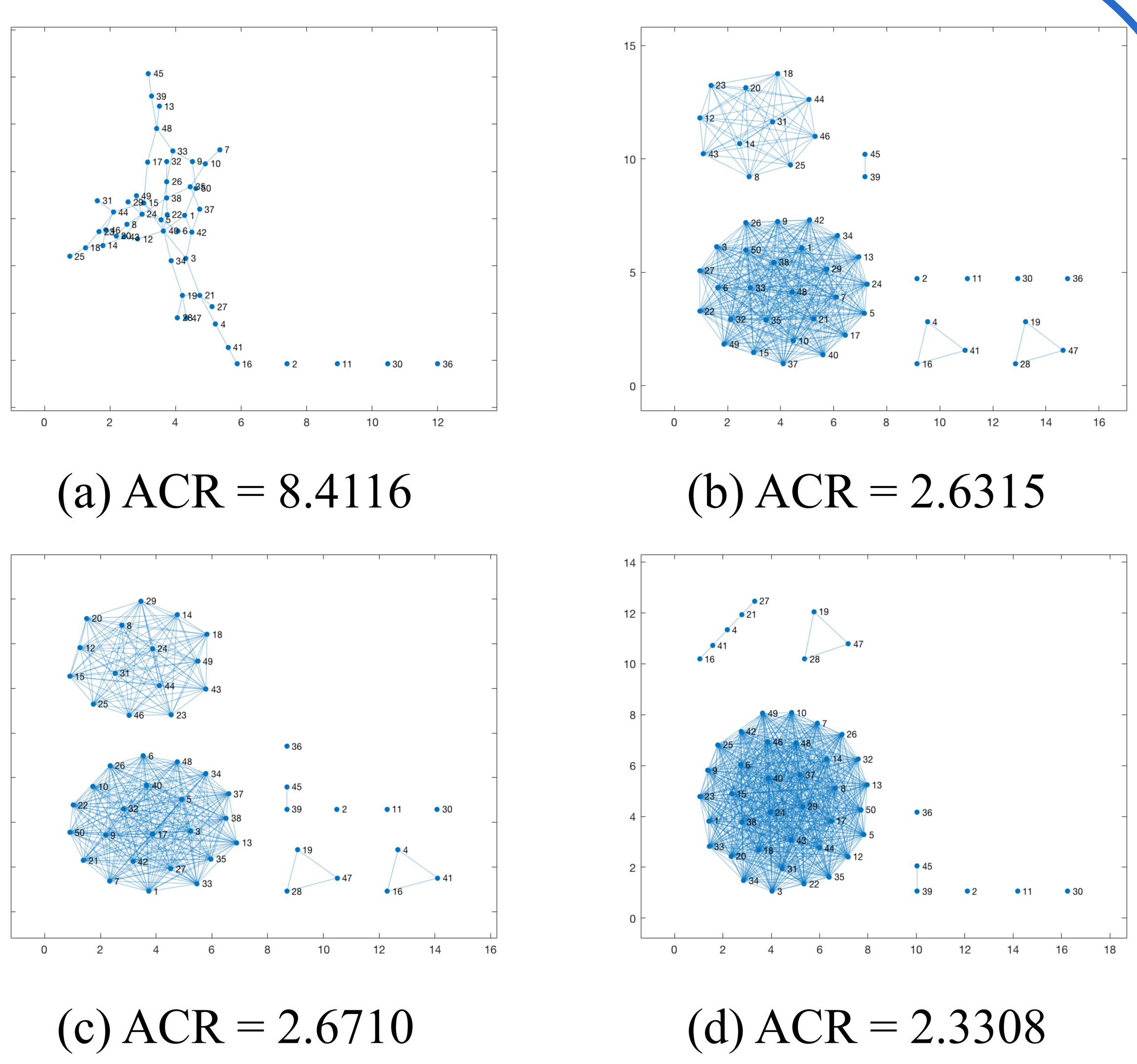


Figure 4: Optimal results using the two algorithms for the ACR of *ec*. (a) is the original graph; (b) is the result of coordinate descent; (c) is the result of conditional gradient descent with $k = 5$ at each iteration; (d) is the result of conditional gradient descent with $k = 25$.

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[2] Xi Chen, Jefrey Lijffijt, and Tijl De Bie. Quantifying and minimizing risk of conflict in social networks. Under review.

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