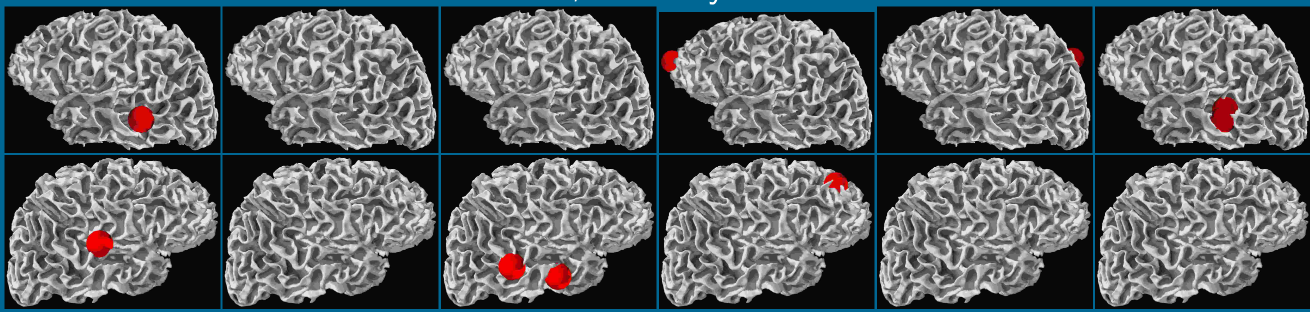
We propose a **convex** concomitant formulation to **jointly** estimate the regression coefficients and the covariance matrix in **high dimensional** linear regression with **correlated Gaussian noise**.

Our estimator outperforms competitors on synthetic and real data.

Real data, auditory stimulation



Concomitant Lasso with repetitions (CLaR)

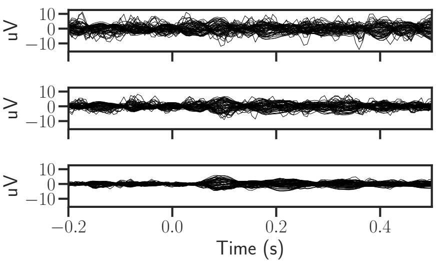
Q. Bertrand[1] M. Massias[1] A. Gramfort[1] J. Salmon[2]. [1]Inria, Univ. Paris Saclay, [2]IMAG, Univ. Montpellier, CNRS

Intro

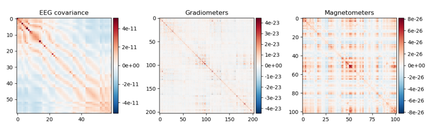
- M/EEG data are **very noisy** (SNR=1)
- it is thus customary to make **several repetitions** of the same experiment
- in order to **average** the signals and increase the signal to noise ratio

Real EEG data

- # of repetitions 5 (top), 10 (middle), 50 (bottom)



- M/EEG data are contaminated with **correlated Gaussian noise**:



Model and notations

Linear Multi-Task setting with correlated Gaussian noise:

- n : # of sensors
- p : # of features
- q : # of tasks/time points
- $X \in \mathbb{R}^{n \times p}$: design matrix
- $\mathrm{B} \in \mathbb{R}^{p \times q}$: regression coefficients
- $S \in \mathbb{R}^{n \times n}$: square root of the covariance matrix
- $\mathrm{E}^{(l)} \in \mathbb{R}^{n \times q}$ random matrix with i.i.d. normal entries
- $Y^{(l)} \in \mathbb{R}^{n \times q}$: signals

Model: $Y^{(l)} = X\mathrm{B}^* + S^*\mathrm{E}^{(l)}$, $\forall l \in [r]$

- $\bar{Y} = \frac{1}{r}\sum_l Y^{(l)} \in \mathbb{R}^{n \times n}$ mean of the signals across repetitions

Our approach: use repetitions

- Concomitant Lasso with Repetitions (CLaR)

$$(\hat{\mathrm{B}}, \hat{S}) \in \mathop{\arg\min}_{\substack{\mathrm{B} \in \mathbb{R}^{p \times q} \\ S \in \mathbb{S}^n_{++}, S \succeq \underline{\sigma}}} \frac{\sum_{l=1}^{r} \left\| Y^{(l)} - X\mathrm{B} \right\|_{S^{-1}}^2}{2nqr} + \frac{\mathrm{Tr}(S)}{2n} + \lambda \|\mathrm{B}\|_{2,1}$$

Previous approaches: use the mean

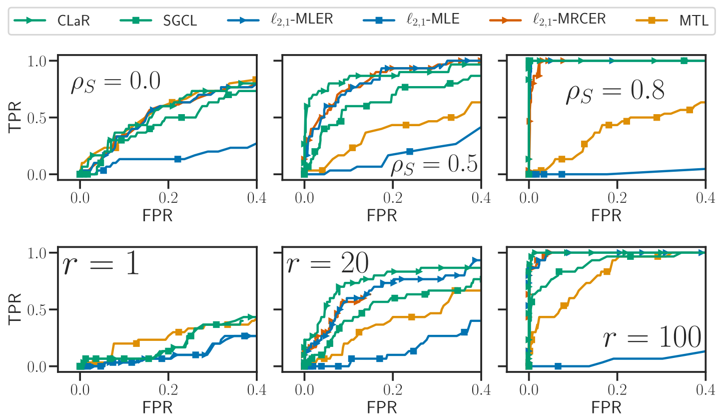
- Multi-Task Lasso (MTL)

$$\hat{\mathrm{B}} \in \mathop{\arg\min}_{\mathrm{B} \in \mathbb{R}^{p \times q}} \frac{1}{2nq} ||\bar{Y} - X\mathrm{B}||^2 + \lambda \|\mathrm{B}\|_{2,1}$$

- SGCL

$$(\hat{\mathrm{B}}, \hat{S}) \in \mathop{\arg\min}_{\substack{\mathrm{B} \in \mathbb{R}^{p \times q} \\ S \in \mathbb{S}^n_{++}, S \succeq \underline{\sigma}}} \frac{||\bar{Y} - X\mathrm{B}||_{S^{-1}}^2}{2nq} + \frac{\mathrm{Tr}(S)}{2n} + \lambda \|\mathrm{B}\|_{2,1}$$

More experiments



Online code: `https://github.com/QB3/CLaR`

Download the paper →