We propose a convex concomitant formulation to jointly estimate the regression coefficients and the covariance matrix in high dimensional linear regression with correlated Gaussian noise.

Our estimator outperforms competitors on synthetic and real data.

**Real data, auditory stimulation**

![Image of brain scans showing activation patterns]

**Concomitant Lasso with repetitions (CLaR)**

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**Intro**
- M/EEG data are very noisy (SNR=1)
- It is thus customary to make several repetitions of the same experiment
- In order to average the signals and increase the signal to noise ratio
- **Real EEG data**
  - # of repetitions 5 (top), 10 (middle), 50 (bottom)

**Model and notations**
- Linear Multi-Task setting with correlated Gaussian noise:
  - \( n \): # of sensors
  - \( p \): # of features
  - \( q \): # of tasks/time points
  - \( X \in \mathbb{R}^{n \times p} \): design matrix
  - \( B \in \mathbb{R}^{p \times q} \): regression coefficients
  - \( S \in \mathbb{R}^{n \times n} \): square root of the covariance matrix
  - \( E(l) \in \mathbb{R}^{n \times q} \): random matrix with i.i.d. normal entries
  - \( y(l)(t) \in \mathbb{R}^{n \times q} \): signals
  - \( \bar{y} = \frac{1}{r} \sum_l y(l) \in \mathbb{R}^{n \times q} \): mean of the signals across repetitions

**Model:**
\[
y(l)(t) = X u^* + S^* e(l)(t), \quad \forall t \in [r]
\]

Our approach: use repetitions
- **Concomitant Lasso with Repetitions (CLaR)**
  \[
  (B, S) \in \arg \min_{B \in \mathbb{R}^{p \times q}, S \in \mathbb{R}^{n \times n}, \Sigma \succeq \sigma} \frac{1}{2nq} \sum_{l=1}^r \| y(l) - X B \|_2^2 - \frac{1}{2nq} \text{Tr}(S^2) - \frac{1}{2nqr} + \lambda \| B \|_{2,1}
  \]

Previous approaches: use the mean
- **Multi-Task Lasso (MTL)**
  \[
  B \in \arg \min_{B \in \mathbb{R}^{p \times q}} \frac{1}{nq} \| \bar{y} - X B \|_2^2 + \lambda \| B \|_{2,1}
  \]
- **SGCL**
  \[
  (B, S) \in \arg \min_{B \in \mathbb{R}^{p \times q}, S \in \mathbb{R}^{n \times n}, \Sigma \succeq \sigma} \frac{1}{2nq} \| \bar{y} - X B \|_2^2 - \frac{1}{2nq} \text{Tr}(S^2) - \frac{1}{2nqr} + \lambda \| B \|_{2,1}
  \]

**More experiments**
- CLaR SGCL \( \ell_{2,1} \)-MLER \( \ell_{2,1} \)-MLE \( \ell_{2,1} \)-MRCER MTL

**Online code:** [https://github.com/QB3/CLaR](https://github.com/QB3/CLaR)

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