

Introduction

In case of an accidental radioactive release, IRSN uses atmospheric dispersion models to evaluate the radiological consequences for the human health and the environment. Dispersion model results highly depend on the accuracy of the meteorological fields and the source term including the location, the duration, the magnitude and the isotopic composition of the release. To improve the source term assessment, IRSN has developed an operational tool based on variational inverse modeling techniques (Saunier et al., 2013; Winiarek et al., 2012) which consists in combining dispersion model and environmental observations. More advanced methods, based on Bayesian inference, were recently applied to improve the Fukushima source term and to quantify the uncertainties associated to each source term parameter (Liu et al., 2017). In this study, Bayesian Monte Carlo Markov Chains (MCMC) methods are applied to reconstruct the source term of the ¹⁰⁶Ru detections event in 2017.

Ruthenium 106 episode

Between late September and mid-October 2017, small amounts of Ruthenium 106 have been detected by European monitoring networks. Ruthenium 106 was measured at levels ranging from a few μBq/m³ to more than 170mBq/m³ in Romania. The location of the source was unknown.

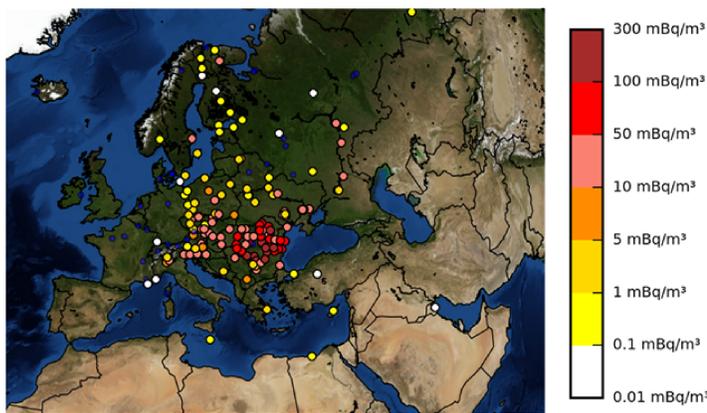


Figure 1: Maximum air concentration measures of Ruthenium 106 observed over Europe in mBq/m³. Air sampling period varies from half a day to one month.

Over Europe, more than a thousand air concentration measures observed amongst 296 different stations are used to retrieve the source term's variables of interest: the longitude, the latitude, the release rates, assumed to be daily, and the observation-prediction error covariance matrix **R**. Location is assumed to be included in the domain of dimensions [6W, 70 E] and [34 N, 68 N].

Bayesian inverse modeling

Bayes' formula applied to the ¹⁰⁶Ru source reconstruction problem can be stated as follows:

$$p(\mathbf{x}|\mathbf{y}) = \frac{p(\mathbf{y}|\mathbf{x})p(\mathbf{x})}{p(\mathbf{y})} \propto p(\mathbf{y}|\mathbf{x})p(\mathbf{x}). \quad (1)$$

where p is the probability distribution, \mathbf{y} the observation vector and \mathbf{x} the source vector variables. The variables joint distribution is depending upon the likelihood of the observations and the priors. Their derivation yield:

$$p(\mathbf{x}|\mathbf{y}) \propto \frac{1}{|\mathbf{R}|^{1/2}} e^{-\frac{1}{2}(\log(\mathbf{y}/\mathbf{H}\mathbf{x}))^T \mathbf{R}^{-1}(\log(\mathbf{y}/\mathbf{H}\mathbf{x}))} + e^{\log\|\frac{\mathbf{q}}{\mathbf{q}_{ref}}\|k} e^{-\log\|\frac{\mathbf{q}}{\mathbf{q}_{ref}}\|/\theta}, \quad (2)$$

where the first term represents the likelihood and the second a prior on the release vector \mathbf{q} . \mathbf{H} is the matrix representing the resolvent of the atmospheric transport model and is computed using IdX atmospheric dispersion model and Météo-France meteorological data. We will use MCMC methods to sample from $p(\mathbf{x}|\mathbf{y})$. ent of the atmospheric transport model and is computed using IdX atmospheric dispersion model and Météo-France meteorological data. We will use MCMC methods to sample from $p(\mathbf{x}|\mathbf{y})$.

References

- Liu, Y., J.-M. Haussaïre, M. Bocquet, Y. Roustan, O. Saunier, and A. Mathieu, 2017: Uncertainty quantification of pollutant source retrieval: comparison of Bayesian methods with application to the Chernobyl and Fukushima Daiichi accidental releases of radionuclides. *Quarterly Journal of the Royal Meteorological Society*, **143** (708), 2886–2901, doi:10.1002/qj.3138.
- Saunier, O., A. Mathieu, D. Didier, M. Tombette, D. Quélo, V. Winiarek, and M. Bocquet, 2013: An inverse modeling method to assess the source term of the Fukushima Nuclear Power Plant accident using gamma dose rate observations. *Atmospheric Chemistry and Physics*, **13** (22), 11403–11421, doi:https://doi.org/10.5194/acp-13-11403-2013.
- Winiarek, V., M. Bocquet, O. Saunier, and A. Mathieu, 2012: Estimation of errors in the inverse modeling of accidental release of atmospheric pollutant: Application to the reconstruction of the cesium-137 and iodine-131 source terms from the Fukushima Daiichi power plant. *Journal of Geophysical Research: Atmospheres*, **117** (D5), doi: 10.1029/2011JD016932.

MCMC methods

Metropolis-Hastings algorithm

A popular MCMC method is the Metropolis-Hastings (MH) algorithm. Once \mathbf{x} is initialised, the algorithm consists in iterating on three steps:

- Generate new candidates \mathbf{x}' from previous state \mathbf{x}_i at iteration i according to some predefined transition probabilities g .
- Compute acceptance ratio α :

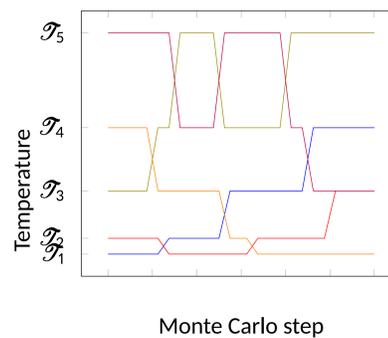
$$\alpha = \frac{p(\mathbf{y}|\mathbf{x}')p(\mathbf{x}')g(\mathbf{x}_i|\mathbf{x}')}{p(\mathbf{y}|\mathbf{x}_i)p(\mathbf{x}_i)g(\mathbf{x}'|\mathbf{x}_i)}. \quad (3)$$

- Accept the proposition if $u \leq \alpha$: accept - set $\mathbf{x}_{i+1} = \mathbf{x}'$ for $u \sim \mathcal{U}(0, 1)$.

Parallel Tempering algorithm

Parallel tempering, an other MCMC algorithm, will be used in order to sample more efficiently on the posterior distribution. The idea of parallel tempering is to combine our MH chain with N replicas initialized at different temperatures $T_N, \dots, T_i, \dots, T_0$ where $T_N > T_{N-1} > \dots > T_0 = 1$. Temperatures > 1 flatten out the target distribution $p(\mathbf{x}|\mathbf{y})$, thus allowing the corresponding chains to explore the entire state space and avoid local minima provided the temperature is small enough. A procedure thereafter swaps configurations between chains at adjacent temperature level.

Parallel tempering switches between two dynamics:



- *Single-temperature move*: each T_i temperature replica performs a simple MH step iteration, attempting to update its current parameters state.
- *Swapping (two chains at adjacent temperatures)*: Swapping between T_i temperature replica and T_j temperature replica is attempted.

Parallel tempering for the source term of the ¹⁰⁶Ru release accident

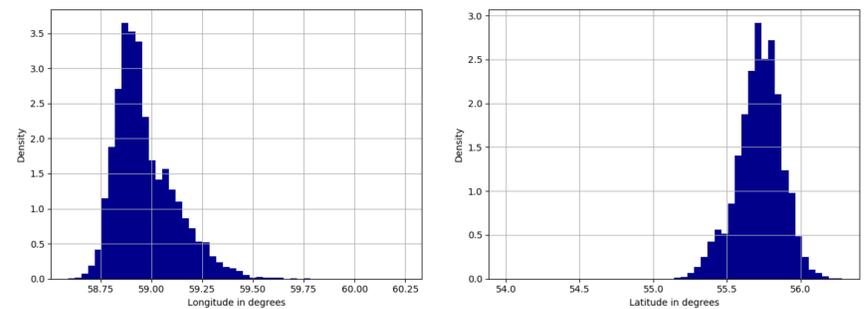


Figure 2: Distributions of the coordinates of the ¹⁰⁶Ru source: Longitude (a), Latitude (b)

- Figure 2 shows that the maximum of the source's longitude and latitude distributions is reached at coordinates [58.5, 56] which corresponds to an area located in southern Ural. Furthermore, the shape of the pdfs states that an other location somewhere else in Europe is unlikely.

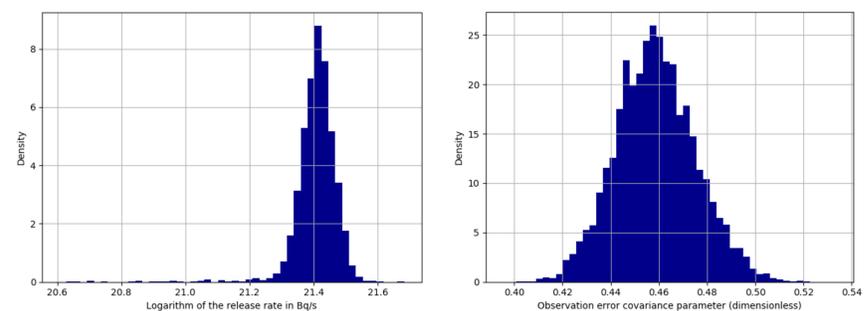


Figure 3: Distributions of the logarithm of the release rate on the 26th of the ¹⁰⁶Ru source and observation error covariance parameter r .

- Results show that Ruthenium has been mainly released the 26th of September with quantities ranging between 100-200 TBq.
- Small model-measurements error (modelling and observation uncertainties).
- A good mixing is achieved according to the evolution of the cost and the nature of the distributions obtained. Simulations quickly converge to an invariant distribution (less than 20 minutes of calculation) and thus are compatible with an operational use.

Conclusion

Advanced Bayesian inverse methods have been applied to identify the Ruthenium 106 source term following the detection event in October 2017. Future distributions will be sampled using several meteorological data sets and types of measure. Our method will then be applied to retrieve the source term of Fukushima.