On Topology Evolution of Temporal Networks
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Background
- A growing interest on graph and graph neural networks in recent years.
- Most of them concentrate on static graphs.
- The very few studies on dynamic graphs tend to focus only on one specific temporal information.

Research Questions
It is a fundamental task in graph mining to predict graph evolution and is barely explored by the current study of graph neural networks.
- What is the "general" temporal information on graph evolution?
- How can we model this information?
- How can we measure the effectiveness of our model?

Objectives
We address the above challenge by
- proposing a model that can capture simultaneously topological and temporal features of dynamic graphs
- using the model to predict the topological evolution of snapshots graphs
- proposing an effective metric to measure the predictive performance

Definition
Dynamic Graph: A sequence of directed or undirected snapshot graphs \( \{G_t\} \), with \( G_t = (V_t, E_t) \). \( V_t \) and \( E_t \) are subsets of \( V \) and \( E \) which represent all the nodes and edges appearing within the time range. \( |V_t| \) and \( |E_t| \) are not necessarily monotone.

Materials
Message-Passing Graph Network (topological information)
- \( x_t = \Theta x_t + \sum_{j \in V_t} x_j h_0(e_{ij}) \) (Aggregation)
- \( q = \text{Set2Set}(X) \) (Readout)

Recurrent Neural Network (temporal information)
- \( \hat{q}_t = \text{LSTM}(\{q_{t-w}, \ldots, q_{t-1}\}) \)

Graph Generative Model
- \( \hat{G} = \arg\max P(\hat{A}|\{\hat{a}_1, \ldots, \hat{a}_t\}|q_t') \)
- \( h_t = \text{LSTM}(a_t-1, h_{t-1}) \), where \( h_0 = \Theta q_t' \)
- \( \hat{a}_t = \sigma (h_t) \)

Methods
Proposed Architecture:
As illustrated, the input graph sequence (length \( w \)) is feed into a graph neural network to extract their topological feature (graph embedding). They are then passed through a recurrent neural network to couple with temporal information. The output at final timestep is considered containing simultaneously topological and temporal information, thus a good representation modelling network dynamics. It is then feed into a graph generator network to generate graph prediction at expected timestep.

Evaluation:
Metric: Graph kernels \( K(G, \hat{G}) \) that measure topological similarity between real graphs and their prediction. (Weisfeiler-Leman Subtree kernel)

Important Results
Datasets:
- Synthetic dataset: Path graph / Ladder graph with evolving size (add or remove nodes)
- Realworld dataset: Bitcoin-Alpha, Bitcoin-OTC, DNC emails graph

Charts below show the comparison between predicted graph sizes (Blue) and real graph sizes (Orange). Left: Path graph; Right: Ladder graph. It seems to be difficult for a relatively complex structure as ladder graph at the beginning but both have good predictions eventually.

References