Privacy Impact on Generalized Nash Equilibrium in Peer-to-Peer Electricity Market

Ilia Shilov[1,2,3], Hélène Le Cadre[2] and Ana Busic[1,3]

Inria Paris (1) / VITO BE (2) / DI ENS, CNRS, PSL University (3)

Introduction

Information sharing in peer-to-peer electricity market can improve agents' performance, but also may violate their privacy. This calls for the design of new communication mechanisms that capture the agents' ability to define the information they want to share with the other market participants, while preserving their privacy. In many applications, this problem is addressed by including a noise to the reports that the agents subsequently use to compute the market equilibrium [1]. However, this approach does not include the ability of the agents to act strategically on the values of their reports. In our proposed framework, agents compute Generalized Nash Equilibrium (GNE) [2] with respect to the constraints that bound (a) the distance between the deterministic deviation from the true values of the private information and (b) the Kullback-Leibler divergence, which measures the effect of the additive random noise included in the reports.

Electricity trading problem

We focus on the privacy issues that arise after solving the peer-to-peer electricity trading problem, formulated as a Generalized Nash Equilibrium Problem considered in [1]. Each prosumer $n \in \mathcal{N}$ solves:

$$\begin{aligned}
\min_{\boldsymbol{x}_n} \quad & \Pi_n(\boldsymbol{x}_n), \\
s.t. \quad & \underline{G}_n \le G_n \le \overline{G}_n && (\underline{\mu}_n, \overline{\mu}_n) \\
& \underline{D}_n \le D_n \le \overline{D}_n && (\underline{\nu}_n, \overline{\nu}_n) \\
& q_{mn} + q_{nm} = 0 && (\zeta_{nm}) \\
& q_{mn} \le \kappa_{mn} && (\xi_{nm}) \\
& D_n = G_n + \Delta G_n + \Sigma_{m \in \Omega_n} q_{mn} && (\lambda_n),
\end{aligned}$$

where $x_n := (D_n, G_n, \boldsymbol{q}_n)$, i.e. each agent chooses the bilateral trades $\boldsymbol{q}_n$ with agents she wants to trade electricity with, self-generation G_n and flexible demand D_n. The objective function of the agent n:

$$\Pi_n(D_n, G_n, \boldsymbol{q}_n) := 1/2 \cdot a_n G_n^2 + b_n G_n + d_n + \tilde{a}_n (D_n - D_n^*)^2 - \tilde{b}_n + \Sigma_{m \in \Omega_n, m \neq n} c_{nm} q_{mn},$$

ΔG_n denotes the *renewable energy sources (RES)-based generation* at node n. Corresponding dual variables are placed in blue at the right of each constraint.

Link between electricity trading and communication game

- **Assumption 1.** *We assume that there are large trading capacities from and to node 0 – that is $\xi_{0n} = \xi_{n0} = 0 \ \forall n \in \mathcal{N}$ and $c_{n0} = c_{0n}$ for all $n \in \mathcal{N}$.*
- Each agent holds the private information $y_n := D_n^* - \Delta G_n$, where D_n^* is the *target demand.*
- At the Variational equilibrium (VE), agent n's decision variables $\boldsymbol{x}_n^*$ depend on the dual variable λ_n [1].
- Under Assumption 1 λ_n are aligned across agents, i.e. $\lambda_n = \lambda_0, \forall n \in \mathcal{N}$, where λ_0 is the *uniform market clearing price*, that depends on y_n [1]:

$$\lambda_0 = \frac{\Sigma_n y_n + \Sigma_n \frac{b_n}{a_n}}{\Sigma_n \left(\frac{1}{2\tilde{a}_n} + \frac{1}{a_n} \right)}$$

- Decision variables D_n, G_n, Q_n are given at the equilibrium by the following expressions:

$$D_n(\boldsymbol{y}) = D_n^* - \frac{1}{2\tilde{a}_n}\lambda_0$$

$$G_n(\boldsymbol{y}) = -\frac{b_n}{a_n} + \frac{1}{a_n}\lambda_0$$

$$Q_n(\boldsymbol{y}) = D_n^* + \frac{b_n}{a_n} - \left(\frac{1}{a_n} + \frac{1}{2\tilde{a}_n}\right)\lambda_0 - \Delta G_n,$$

where $a_n, b_n, d_n, \tilde{a}_n, \tilde{b}_n > 0$ are prosumer n's production and consumption cost parameters.

Reports of the agents

- The report of the agent n takes the form:

$$\tilde{y}_n = \hat{y}_n + \varepsilon_n$$

- Define an upper bounded distance as a symmetric *adjacency relation* $y_n \simeq \hat{y}_n$ for agent n:

$$y_n \simeq \hat{y}_n \iff d(y_n, \hat{y}_n) \le \alpha_n,$$

where α_n is chosen beforehand.

- Agent n samples a Gaussian noise $\varepsilon_n \sim \mathcal{N}(0, \sigma_n^2)$ (with $\sigma_n > 0$), giving rise to the report $\tilde{y}_n \sim \mathcal{N}(\hat{y}_n, \sigma_n^2)$.

Communication game

Denote $V_n := \sigma_n^2$. To decide on the optimal value of the report $\tilde{y}_n$, each agent needs to solve the following optimization problem:

$$\begin{aligned}
\min_{\hat{y}_n, V_n} \quad & \mathbb{E}_{\varepsilon_n \sim \mathcal{N}(0,V_n)} \Big(\Pi_n(\hat{\boldsymbol{y}}, \boldsymbol{\varepsilon}) \Big) \\
s.t. \quad & \underline{G}'_n \le \mathbb{E}_{\varepsilon_n \sim \mathcal{N}(0,V_n)} (G_n(\tilde{\boldsymbol{y}})) \le \overline{G}'_n && (\underline{\mu}_n, \overline{\mu}_n) \\
& \underline{D}'_n \le \mathbb{E}_{\varepsilon_n \sim \mathcal{N}(0,V_n)} (D_n(\tilde{\boldsymbol{y}})) \le \overline{D}'_n && (\underline{\nu}_n, \overline{\nu}_n) \\
& (\hat{y}_n - y_n)^2 \le \alpha_n^2 && (\underline{\gamma}_n, \overline{\gamma}_n) \\
& D_{KL}(M(y_n) \,||\, M(\hat{y}_n)) \le A_n && (\underline{\beta}_n, \overline{\beta}_n),
\end{aligned}$$

where $M(\cdot) = \cdot + \varepsilon_n$ and $D_{KL}(M(y_n)||M(\hat{y}_n))$ is the *Kullback-Leibler divergence (or the relative entropy)* between M's output distributions on y_n and $\hat{y}_n$.

GNE analysis

Since λ_0 depends on the sum of $\Sigma_n y_n$, the communication game has an *aggregative game* structure.

- **Proposition 1.** *Operator*

$$\boldsymbol{F}(\hat{\boldsymbol{y}}, \boldsymbol{V}) := \left(\nabla_n \mathbb{E}(\Pi_n(\hat{\boldsymbol{y}}, \boldsymbol{V})) \right)_{n=1}^N$$

for the communication game is strongly monotone. It follows that Variational equilibrium solution of the communication game is unique [2].

- Denote $B_n := (\frac{1}{a_n} + \frac{1}{2\tilde{a}_n})$ and $B = \Sigma_n B_n$.
- For the variance, from the KKT conditions we obtain that for all $n \in \mathcal{N}$:

$$V_n = \frac{2B^4}{A_n B_n^2} (\overline{\beta}_n + \underline{\beta}_n)^2 \tag{2}$$

- **Proposition 2.** *Dual variables $\underline{\beta}_n, \overline{\beta}_n$ denote the privacy price for agent n and are computed by the formula*

$$(\overline{\beta}_n + \underline{\beta}_n)^2 = \frac{B_n^2 (\hat{y}_n - y_n)^2}{4B^4}$$

- **Assumption 2.** *We assume that $\frac{B_n}{B} \simeq \frac{B_m}{B}$, $n \neq m$ for all $n, m \in \mathcal{N}$ i.e. each agent n's contribution B_n to the sum B is small.*

  Proposition 3. *Under Assumption 2, the communication game is a Generalized Potential Game.*

Algorithm and numerical results

We consider the IEEE 14-bus network system, where each bus of the network corresponds to a prosumer in our model. We employ the penalized individual cost functions to deal with coupled constraints and use stochastic approximation gradient-based scheme to approach a GNE.

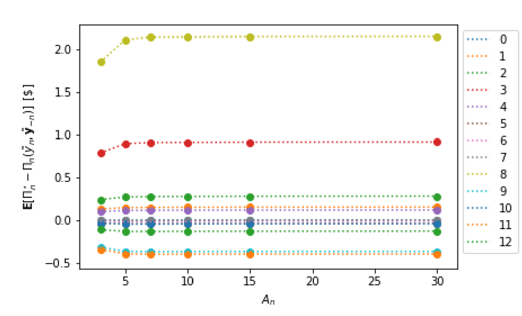

Fig. 1:Utility gap wrt. A_n

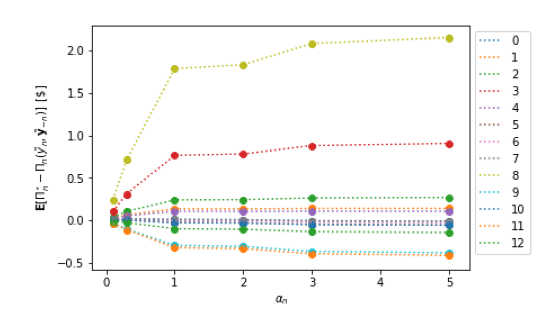

Fig. 2:Utility gap wrt. α_n

Figures 3 and 4 depicts the dependance of the social cost of the system with respect to A_n and α_n.

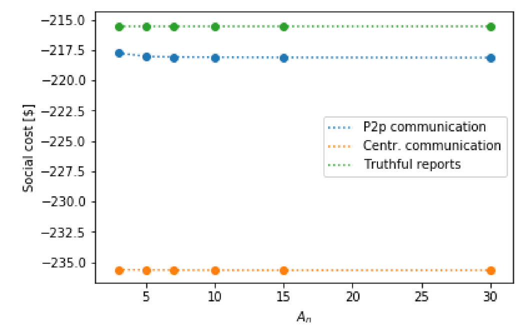

Fig. 3:Social cost wrt. A_n

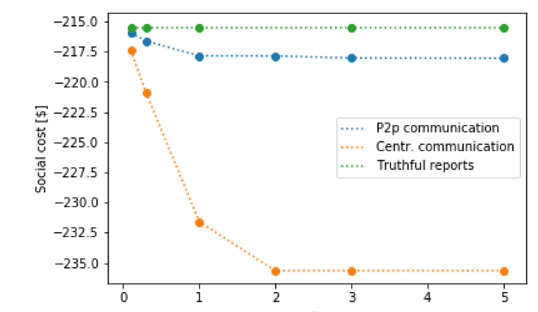

Fig. 4:Social cost wrt. α_n

We compare three instances: (a) peer-to-peer communication mechanism, (b) fully coordinated communication mechanism and (c) the social cost evaluated in the truthful reports.

References

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[2] A. A. Kulkarni and U. V. Shanbhag, "On the variational equilibrium as a refinement of the Generalized Nash equilibrium", *Automatica*, vol. 48, no. 1, pp. 45–55, 2012.